THE EFFECT OF DYNAMIC FLUID PRESSURE ON A DAM DURING EARTHQUAKES

(O DINAMICHESKOM DAVLENII ZHIDKOSTI NA PLOTINU PRI ZEMLETRIASENII)

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When working out the effect of earthquakes on hydraulic dams only the stresses caused by the free vibrations of the dam are taken into account [1]. It is evident, however, that when there is an earthquake the water exerts an additional dynamic pressure on a dam. An attempt has been made to take this into account in [2] and [3], but the boundary conditions have only been satisfied approximately, i.e. the surface wave on the liquid has been neglected, and the displacement of the dam has not been taken into account in satisfying the boundary conditions on it. In this article the problem of determining the dynamic pressure is handled with greater precision. It is demonstrated that the additional dynamic pressure cannot be neglected, for in some cases it exceeds the hydrostatic pressure, especially in the upper sections of the dam.

We assume that a dam is located in the plane $x = U_0 \sin \omega t$ in a rectangular coordinate system x, y, z. A portion of the space, bounded by $x \geqslant U_0 \sin \omega t - h \leqslant y \leqslant 0, -\infty \leqslant z \leqslant \infty$, is filled with liquid. We will deal with the wave motion and the dynamic fluid pressure caused by an instantaneous initial velocity V_0 acquired by the dam as a result of an earthquake, i.e. a velocity V_0 is induced impulsively on the fluid by the dam at $x = 0$, $t = 0$. Then the dam oscillates according to $V = V_0 \cos \omega t$.

It is evident that in the study of liquid waves and dynamic pressures arising therefrom, the influence of the deflection of the dam is negligibly small. If, therefore, we denote the velocity potential in the fluid as $\phi(x, y, t)$ and assume the liquid incompressible and its free surface to be the first to come to rest, we can state the initial conditions of the problem thus:

$$\frac{\partial \varphi (x, 0, 0)}{\partial t} = 0, \qquad \frac{\partial \varphi (0, y, 0)}{\partial x} = V_0 \tag{1}$$

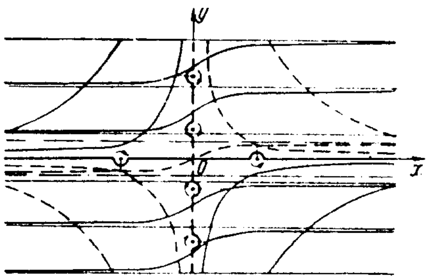

Fig. 1.

and the boundary conditions

$$\frac{\partial\varphi}{\partial x} = V_0 \cos\omega t \quad \text{when } x = U_0 \sin\omega t \qquad (2)$$

$$\frac{\partial\varphi}{\partial y} = 0 \quad \text{when } y = -h$$

$$\frac{\partial^2\varphi}{\partial t^2} + g\frac{\partial\varphi}{\partial y} = 0 \quad \text{when } y = 0 \qquad (3)$$

where U_0, V_0 are displacement and velocity amplitudes of the dam vibration, respectively.

The velocity potential of the required solution should satisfy the Laplace equation $\Delta\phi = 0$ and also the above conditions. We assume the velocity potential $\phi(x, y, t)$ to satisfy the Laplace equation in this form

$$\varphi(x, y, t) = \cos\omega t \left\{ \int_0^\infty [A(\alpha)\cos\alpha y + B(\alpha)\sin\alpha y]\, e^{-\alpha X}\, d\alpha + \right.$$

$$\left. + \iint_0^\infty [C(\alpha, k)\cosh k(y+h) + D(\alpha, k)\sinh ky]\cos kX\, d\alpha\, dk \right\} \qquad (4)$$

In this expression

$$X = x - U_0 \sin\omega t$$

The arbitrary functions $A(\alpha)$, $B(\alpha)$, $C(\alpha, k)$, $D(\alpha, k)$ are determined from the boundary and initial conditions. Let us assume that

$$\varphi = \varphi(x, y, t) \ \text{when } x \geqslant U_0 \sin\omega t, \qquad \varphi = 0 \ \text{when } x < U_0 \sin\omega t \qquad (5)$$

If we insert $\phi(x, y, t)$ into the second boundary condition (3) we obtain

$$QA(\alpha) = \alpha B(\alpha) \quad (Q = \omega^2/g) \quad (kg\sinh kh - \omega^2\cosh kh)\, C(\alpha, k) = -kgD(\alpha, k) \qquad (6)$$

and on satisfying condition (2) we obtain the integral equation

$$-\int_0^\infty \frac{B(\alpha)}{Q}[\alpha\cos\alpha y + Q\sin\alpha y]\,\alpha\, d\alpha = V_0 \qquad (7)$$

Introduce new variables $\lambda = (\alpha/Q)^2$, $y = -\nu$ and rewrite Equation (7) thus:

$$-\frac{\sqrt{2}}{\pi}\int_0^\infty \frac{\pi B(Q\sqrt{\lambda})\, Q^2(\lambda+1)}{2\sqrt{2}}\left[\cos\sqrt{\lambda}Q\nu - \frac{\sin\sqrt{\lambda}Q\nu}{\sqrt{\lambda}}\right]\frac{\sqrt{\lambda}}{\lambda+1}\, d\lambda = V_0\psi(\nu) \qquad (8)$$

where

$$\psi(\nu) = 0 \ \textbf{when}\ h < \nu < \infty \ \textbf{and}\ -\infty < \nu < 0; \qquad \psi(\nu) = 1 \ \textbf{when}\ 0 \leqslant \nu \leqslant h \tag{9}$$

Using the well-known transformation

$$F_0(\lambda) = \int_0^\infty f_0(\varkappa)\left[\sin\theta\cos\sqrt{\lambda}\varkappa - \cos\theta\,\frac{\sin\sqrt{\lambda}\varkappa}{\sqrt{\lambda}}\right]d\varkappa$$

$$f_0(\varkappa) = \frac{1}{\pi}\int_0^\infty F_0(\lambda)\left[\sin\theta\cos\sqrt{\lambda}\varkappa - \cos\theta\,\frac{\sin\sqrt{\lambda}\varkappa}{\sqrt{\lambda}}\right]\frac{\sqrt{\lambda}\,d\lambda}{\lambda\sin^2\theta + \cos^2\theta} \tag{10}$$

with $\theta = \pi/4$, and bearing in mind the nature of the function $\psi(\nu)$ we determine the arbitrary function

$$B(\alpha) = \frac{2V_0 Q\,[Q(1-\cos\alpha h) - \alpha\sin\alpha h]}{\pi\alpha^2(\alpha^2+Q^2)} \tag{11}$$

On satisfying the first condition (3) we arrive at a different integral equation

$$\int_0^\infty\left[\int_0^\infty D(\alpha,\ k)\,k\cosh kh\cos kX\,dk + \frac{2V_0}{\pi}G(\alpha)M(\alpha)e^{-\alpha X}\right]d\alpha = 0 \tag{12}$$

where

$$G(\alpha) = \pi B(\alpha)/2V_0 Q, \qquad M(\alpha) = \alpha(\alpha\sin\alpha h + Q\cos\alpha h)$$

Assuming the expression in square brackets to vanish, we introduce the new variable $\xi = X$; bearing in mind the character of the function $\phi(x,\ y,\ t)$ and using the Fourier integral formula we obtain

$$\frac{\pi}{2}D(\alpha,\ k)\,k\cosh kh + \frac{2V_0}{\pi}G(\alpha)M(\alpha)\int_0^\infty e^{-\alpha\xi}\cos k\xi\,d\xi = 0$$

From this we have

$$D(\alpha,\ k) = -\frac{4V_0\alpha^2(\alpha\sin\alpha h + Q\cos\alpha h)}{\pi^2 k(\alpha^2+k^2)\cosh kh}G(\alpha) \tag{13}$$

Now that all the arbitrary coefficients in (4) are determined and the velocity potential obtained, $\phi(x,\ y,\ t)$ satisfies the initial conditions (1).

If we differentiate the velocity potential $\phi(x,\ y,\ t)$ with respect to t, at $x = U_0\sin\omega t$ we obtain the following:

$$\frac{\partial\varphi}{\partial t} = \frac{2V_0}{\pi}\,\omega\sin\omega t\left\{\frac{2}{\pi}\iint_0^\infty\frac{f(\alpha)\sinh ky\,d\alpha\,dk}{k(\alpha^2+Q^2)(\alpha^2+k^2)\cosh kh} - \int_0^\infty\frac{F(\alpha,\ y)}{\alpha^2(\alpha^2+Q^2)}\,d\alpha - \right.$$

$$-\frac{2g}{\pi}\iint\limits_0^\infty \frac{f(\alpha)\cosh k(y+h)\,d\alpha\,dk}{(\alpha^2+Q^2)(\alpha^2+k^2)(kg\sinh kh-\omega^2\cosh kh)\cosh kh}\Big\} + \frac{2V_0^2}{\pi}\cos^2\omega t\int\limits_0^\infty \frac{F(\alpha,y)\,d\alpha}{\alpha(\alpha^2+Q^2)} \quad (14)$$

where

$$N(\alpha) = [Q(1-\cos\alpha h)-\alpha\sin\alpha h], \qquad F(\alpha, y) = N(\alpha)(\alpha\cos\alpha y + Q\sin\alpha y)$$
$$f(\alpha) = N(\alpha)M(\alpha)/\alpha$$

In working out the double integral (14) use is made of the theorem of residues. We first of all integrate with respect to α, and we thus have poles on the complex plane lying on the imaginary axis at $\pm iQ$ and $\pm ik$. Then on integrating with respect to k we have poles lying on the real axis at $\pm Q$ and for $\cosh kh = 0$; on the imaginary axis at $\pm im\pi/2h$, where $m = 1, 3, 5, \ldots \infty$, and for the transcendental equation $kg\sinh kh - \omega^2\cosh kh = 0$, we have two roots lying on the real axis at $\pm\gamma_s$ and an infinite number of roots $\pm i\gamma_n$, where $n = 1, 2, 3, \ldots$, along the imaginary axis. These can be found, using Fig. 1 as a guide, by successive approximation to any desired degree of accuracy. We solve the equation for the real roots $\gamma\tanh\gamma = Qh$ and for the imaginary roots $\gamma\tanh\gamma = -QH$, where $\gamma = kh$.

Now, going to the calculations, we have

$$\int\limits_0^\infty \frac{N(\alpha)(Q\cos\alpha h+\alpha\sin\alpha h)}{(\alpha^2+Q^2)(\alpha^2+k^2)}\,d\alpha = \frac{Q\pi R_0}{k^2-Q^2} + \frac{\pi}{2}\,\frac{[Q(1-\cosh kh)+k\sinh kh]}{k(Q-k)}\,e^{-kh} \quad (15)$$

and therefore

$$\iint\limits_0^\infty \frac{f(\alpha)\sinh ky\,d\alpha\,dk}{k(\alpha^2+Q^2)(\alpha^2+k^2)\cosh kh} = \frac{\pi}{2}\int\limits_0^\infty \frac{[Q(1-\cosh kh)+k\sinh kh]\,e^{-kh}}{k^2(Q-k)\cosh kh}\sinh ky\,dk +$$
$$+ Q\pi R_0\int\limits_0^\infty \frac{\sinh ky\,dk}{(k^2-Q^2)\,k\cosh kh} = \frac{\pi}{2}\int\limits_0^\infty \frac{f(k)\sin ky\,dk}{k^2(k^2+Q^2)\cos kh} - i\,\frac{\pi^2}{2}\,\frac{R_0\sinh Qy}{Q\cosh Qh} - \quad (16)$$
$$-\frac{\pi^2}{2}\sum_{m=1,3}^\infty \frac{(Q-C\sin C_\pi)(Q+iC)}{hC^2(Q^2+C^2)}\sin Cy - Q\pi^2 R_0\sum_{m=1,3}^\infty \frac{\sin Cy\sin C_\pi}{hC(Q^2+C^2)} + iH$$

in these expressions

$$f(k) = N(k)(Q\cos kh + k\sin kh), \qquad N(k) = [Q(1-\cos kh) - k\sin kh]$$
$$R_0 = (1-e^{-Qh})\,e^{-Qh}, \quad C = m\pi/2h, \quad C_\pi = m\pi/2$$

and also

$$H = \int\limits_0^\infty \frac{N(k)(k\cos kh - Q\sin kh)}{k^2(k^2+Q^2)\cos kh}\sin ky\,dk \quad (17)$$

If we work out the second double integral in Formula (14) in the same manner and substitute into it the results of the integration, we obtain for the case $x = U_0 \sin \omega t$

$$\frac{\partial\varphi}{\partial t} = -2V_0Qh^2\omega \sum_{n=1}^{\infty} \frac{C_n \cos[\gamma_n (y+h)/h] \sin \omega t}{[\gamma_n^2 - (1-Qh)\,Qh]\,\gamma_n \cos\gamma_n} - V_0^2 [1 - 2(1-e^{-Qh})\,e^{Qy}] \cos^2 \omega t \tag{18}$$

where

$$C_n = 1 - C_n', \quad C_n' = 2R_0\gamma_n^2 / (\gamma_n^2 + Q^2h^2)\cos\gamma_n$$

We then integrate $\phi(x, y, t)$ with respect to y; for the case $x = U_0 \sin \omega t$ we have

$$\frac{\partial\varphi}{\partial y} = \frac{2V_0}{\pi}\cos\omega t \left\{ \int_0^{\infty} \frac{N(\alpha)\,[Q\cos\alpha y - \alpha \sin\alpha y]}{\alpha(\alpha^2+Q^2)}\,d\alpha - \frac{2}{\pi}\iint_0^{\infty} \frac{f(\alpha)\cosh ky\,d\alpha\,dk}{(\alpha^2+Q^2)(\alpha^2+k^2)\cosh kh} + \right.$$
$$\left. + \frac{2g}{\pi}\iint_0^{\infty} \frac{f(\alpha)\,k\sinh k(y+h)\,d\alpha\,dk}{(\alpha^2+Q^2)(\alpha^2+k^2)(kg\sinh kh - \omega^2\cosh kh)\cosh kh} \right\} \tag{19}$$

Here, on carrying out similar calculations for $x = U_0 \sin \omega t$ we have

$$\frac{\partial\varphi}{\partial y} = -2V_0Qh \sum_{n=1}^{\infty} \frac{C_n \sin[\gamma_n (y+h)/h]}{[\gamma_n^2 - (1-Qh)\,Qh]\cos\gamma_n} \cos\omega t \tag{20}$$

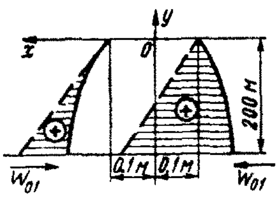

Fig. 2.

It is known that

$$\frac{\partial\varphi}{\partial x} = V_0 \cos\omega t \tag{21}$$

Denoting the liquid density by ρ and the dynamic pressure by P^*, we will have the formula

$$\frac{P^*}{\rho} = -\frac{\partial\varphi}{\partial t} - \frac{1}{2}\left[\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2\right] \tag{22}$$

Let us transform Formula (18) to the following form:

$$\frac{\partial\varphi}{\partial t} = -2U_0 g \sum_{n=1}^{\infty} \frac{C_n \cos[\gamma_n (y+h)/h]}{\gamma_n [R_1 - (R_2 - 1)]\cos\gamma_n} \sin\omega t - V_0^2 [1 - 2(1-e^{-Qh})\,e^{Qy}] \cos^2\omega t \tag{23}$$

$$(R_1 = (\gamma_n / Qh)^2, \ R_2 = 1/Qh)$$

Introduce the following notations: $\phi_y = \partial\phi/\partial y$, $\phi_t = \partial\phi/\partial t$, (for $x = U_0 \sin \omega t$). On combining (20) and (18) it is evident that if ωh is large ϕ_y is a quantity of higher order. From the first term of the series we see that for a given amplitude of displacement U_0 with increase in h or ω, the dynamic pressure in the liquid increases considerably because in this case $\gamma_n \to 1/2\,\pi$ (Fig. 1) and $\cos\gamma_n \to 0$, while $C_n \to 1$, $(R_1 - (R_2 - 1)) \to 1$. It follows from this that the numerator maintains some finite value while the denominator decreases rapidly, and thus the value of ϕ_t increases rapidly. It should be noted that $\gamma_n \to n\pi$ as n increases; therefore $\cos \gamma_n \to 1$ (Fig. 1), and therefore the series which is contained in Formula (23) converges rapidly. This allows us to obtain the distribution of dynamic pressure P^* along the dam easily. As an example, taking the first two terms in the summation in (23), we find a maximum value at the bottom of the dam at a fluid depth of 200 m.

1) $\phi_t = 96720U_0$ (for $\omega = 20$), if $U_0 \geqslant 0.021$ m the dynamic pressure $P^* = 2031\rho$ is greater than the static pressure $P^\circ = 1960\rho$.

2) $\phi_t = 11502U_0$ (for $\omega = 10$), if $U_0 \geqslant 0.18$ m, $P^* = 2070\rho$.

3) $\phi_t = 5756U_0$ ($\omega = 6$), if $U_0 \geqslant 0.35$ m, $P^* \geqslant 2015\rho$.

If we compare (1) and (2) we see that by doubling ω, ϕ_t increases more than eightfold.

We know from seismic laws that the last case approaches a destructive earthquake. Obviously, the supplementary dynamic pressure exceeds static because $\beta = 103\%$, 150% and 158% when $y = -h$, $-0.5h$ and $-0.1h$, respectively, where β denotes the nondimensional parameter P^*/P°.

In the second case, with $U_0 = 0.10$ m, we approach the case of an intense earthquake and we have $\beta = 59\%$ ($y = -h$), $\beta = 84\%$ ($y = -0.5h$), $\beta = 92\%$ ($y = 0.1h$). With an increase in depth of liquid to 300 m we will have $\phi_t = 13470U_0$ ($\omega = 10$) and $\phi_t = 6740U_0$ ($\omega = 6$). It is clear from this analysis that with a destructive earthquake and a strong earthquake the dynamic pressure of the liquid, caused by the vibration of the dam, exerts a great influence on the loading, especially at the higher dam sections.

If we substitute (18), (20) and (24) into Formula (22) we see that the maximum pressure appears at the instant at which the vibrating dam, on reaching $x = -U_0$, assumes maximum acceleration and starts moving in the opposite direction, i.e. to meet the direction of motion of the liquid, while the minimum pressure arises when the dam, on reaching $x = U_0$, moves at maximum acceleration away from the liquid.

Given values of h, ω, U_0 and V_0, using Formulas (18) to (22), it is easy to construct graphs of the maximum fluid-pressure distribution along the dam. For example, for the case $h = 200$ m, $\omega = 10$, $U_0 = 0.1$ m and $\omega = 6$, $U_0 = 0.36$ m, we have constructed the graphs shown on Figs. 2 and 3, respectively, the broken line representing static pressure and the full line the supplementary dynamic pressure; W_{01}, W_{02} are the maximum dam accelerations. In Fig. 3 it is evident that for this case with $x = U_0$ the dam is subject to negative loading amounting to 4500 and 5500 kg/m^2

(with $y = -200$ and -10 m, respectively), but this is less than atmospheric pressure, 1×10^4 kg/m^2.

In order to avoid the development of destructive pressures in the liquid, or to avoid their maxima exceeding a certain limit set by the design, it is possible, for a given ω, to lower the liquid depth h so that γ_n no longer tends to $\pi/2$ but remains at some value corresponding to our predetermined value. The relation between γ_n and ω and h can be obtained from the equation $\gamma_n \tan \gamma_n = -Qh$ to the requisite degree of accuracy.

We now turn our attention to the liquid waves. The equation which determines the form or shape of the wave surface is as follows:

$$\zeta(x, t) = -\frac{1}{g}\frac{\partial \varphi(x, 0, t)}{\partial t} = \frac{4V_0}{\pi^2}\int\limits_0^{\infty}\!\!\int \frac{\omega f(\alpha)\cos kX \sin \omega t \, d\alpha \, dk}{(\alpha^2 + Q^2)(\alpha^2 + k^2)(kg \sinh kh - \omega^2 \cosh kh)} + \frac{2V_0}{g\pi}\omega \sin \omega t \int\limits_0^{\infty} \frac{N(\alpha) e^{-\alpha X}}{\alpha(\alpha^2 + Q^2)} d\alpha \tag{24}$$

Denoting the double integral by η and integrating first of all with respect to α, we obtain relation (15). Inserting (15) into (24) we arrive at

$$\eta = \frac{Q\pi R_0}{8i}\int\limits_{-\infty}^{\infty} \frac{J(k, x, t)\, dk}{(k^2 - Q^2)(kg \sinh kh - \omega^2 \cosh kh)} - \tag{25}$$

$$-\frac{\pi}{8i}\int\limits_0^{\infty} \frac{[Q(1 - \cosh kh) + k \sinh kh]\, e^{-kh} J(k, x, t)}{k(k - Q)(kg \sinh kh - \omega^2 \cosh kh)}\, dk$$

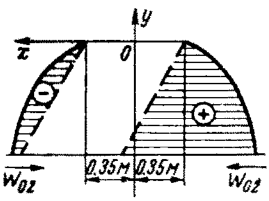

Fig. 3.

where

$$J(k, x, t) = \exp[i(\omega t + kX)] + \exp[i(\omega t - kX)] - \exp[-i(\omega t - kX)] - \exp[-i(\omega t + kX)]$$

On choosing the paths of integration shown in Fig. 1, where the thick full line is the path of the first integration (25) while the broken line is the path of the second integration, and the contours around the poles for the first components of $J(k, x, t)$ are shown in full line and for the latter two components indicated by arrows (this choice of integration path avoids the wave going off from the dam to infinity [5]), on making use of the theory of residues, we finally work out η. On substituting these results of integration in (24) we obtain (for the case $t > 0$)

$$\zeta(x, t) = 2U_0 Q^2 h^2 \sum_{n=1}^{\infty} \frac{C_n \exp(-\gamma_n X/h) \sin \omega t}{\gamma_n[\gamma_n^2 - (1 - Qh)Qh]} + \frac{2U_0 Q^2 h^2 C_s \cos[(\gamma_s/h)(X - \omega h t/\gamma_s)]}{(\gamma_s - Qh)[\gamma_s^2 + (1 - Qh)Qh] \cosh \gamma_s} \tag{26}$$

where $C_s = e^{-\gamma_s} - C_s'$, $C_s' = 2R_0\gamma_s\ (\gamma_s + Qh)$. Formula (26) demonstrates that immediately following the dam motion and close to it the level of the liquid rises and falls in a periodic manner, so that the advancing waves travel from the dam to infinity at a velocity of $\omega h/\gamma_s$, the wave-length being $2\pi h/\gamma_s$.

Note. When determining the arbitrary function $D(\alpha, k)$ the function within the integral sign in (12) with respect to α was assumed zero. We know that this function is aperiodic (the argument α varies between 0 and ∞), and therefore despite the fact that the integral with respect to α is identically zero, the integrand need not be zero. Thus, let us represent the second term in (4) as a single-valued integral in variable k. On satisfying the previous conditions we arrive at functions $D(k)$ and $C(k)$, which will be functions of k multiplied by some constants which, themselves, are integrals in α. For instance,

$$D(k) = I(k)\int_0^\infty L(\alpha)\,d\alpha$$

If we insert these coefficients into $\phi(x, y, z)$ we obtain the previous result, therefore in this case the integrand is equal to zero.

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